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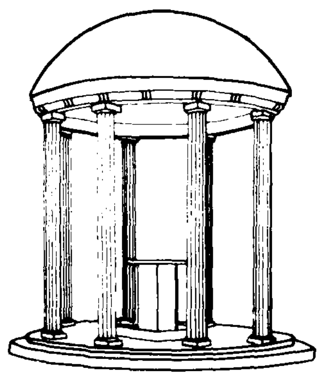

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(9) Technical ~~Report #18~~

(10) Charles Mosier Harvey M. Wagner
Richard Ehrhardt

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Decision Control Models in Operations Research

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This investigation reformulates an approximately optimal algorithm for computing (s,S) inventory policies. The approximation is for a single-item, periodic review model with set-up cost, linear holding and shortage cost, fixed replenishment lead time, and backlogging of unfilled demand.

The analysis repeats the numerical analysis process performed to derive the Power Approximation - an approximately optimal (s,S) policy rule. The

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) research reported here is a modification of the previous derivation to correct for both the non-homogeneity of the Power Approximation and the limiting behavior of the approximation for S-s.

The operating characteristics of the modification are nearly as close to optimal as those of the original Power Approximation.

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FOREWORD

As part of the on-going program in "Decision Control Models in Operations Research," Mr. Charles Mosier has refined the Power Approximation for computing approximately optimal (s,S) inventory policies. Using the approach adopted by Richard Ehrhardt (Technical Report #7), Mr. Mosier uses regression analysis to improve the accuracy of an analytically derived approximation. Mr. Mosier refines the Power Approximation by constraining the regressions to provide a policy that (1) is homogeneous in the units chosen for demand, and (2) has reasonable limiting behavior when the variance of demand approaches zero. The improvements are obtained with only modest sacrifices in total cost performance. Other related reports dealing with this research program are given on the following pages.

Harvey M. Wagner
Principal Investigator

Richard Ehrhardt
Co-Principal Investigator

MacCormick, A. (1974), Statistical Problems in Inventory Control, ONR and ARO Technical Report 2, December 1974, School of Organization and Management, Yale University, 244 pp.

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Ehrhardt, R. and G. Kastner (1980), An Empirical Comparison of Two Approximately Optimal (s,S) Inventory Policies, Technical Report 16, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 22 pp.

Ehrhardt, R. (1980), (s,S) Policies for a Dynamic Inventory Model with Stochastic Lead Times, Technical Report 17, December 1980, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 20 pp.

REVISED (s,S) POWER APPROXIMATION

Charles Mosier*

- Abstract -

This investigation reformulates an approximately optimal algorithm for computing (s,S) inventory policies. The approximation is for a single-item, periodic review model with set-up cost, linear holding and shortage costs, fixed replenishment lead time, and backlogging of unfilled demand.

The analysis repeats the numerical analysis process performed to derive the Power Approximation - an approximately optimal (s,S) policy rule. The research reported here is a modification of the previous derivation to correct for both the non-homogeneity of the Power Approximation and the limiting behavior of the approximation for S-s.

The operating characteristics of the modification are nearly as close to optimal as those of the original Power Approximation.

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1. INTRODUCTION AND SUMMARY

1.1 The Model

We consider a periodic review, single-item inventory system where unfilled demand is backlogged, there is a fixed lead time L between placement and delivery of an order, and demands during review periods are independently and identically distributed, having mean μ and variance σ^2. Replenishment costs are comprised of a setup cost K and a unit cost c. At the end of each review period a cost h or p is incurred for each unit on hand or backlogged, respectively. The criterion of optimality is minimization of the undiscounted expected cost per period over an infinite horizon.

Under these assumptions, an (s,S) policy is optimal (Iglehart [2]), whenever inventory on hand or backlogged plus on order y is less than or equal to s, an order $S-y$ is placed. Algorithmic methods for computing optimal policies are available (Veinott and Wagner [7]), but unfortunately, the computational effort required is prohibitive for practical implementation. Furthermore, the computation of an optimal policy requires the complete specification of the demand distribution, which is an unrealistic assumption in practical settings. Most managers, at best, have imprecise knowledge of the mean and variance of the demand distribution. In response to these practical limitations in available demand information, the Power Approximation was developed by Ehrhardt [1]. The next section describes this approximation.

1.2 The Power Approximation

The Power Approximation computes approximately optimal values for (s,S) using only the mean μ and variance σ^2 of demand. Let $\mu_L = (L+1)\mu$ and $\sigma_L^2 = (L+1)\sigma^2$. The algorithm computes

$$D_p = 1.463\mu^{.364}(K/h)^{.498}\sigma_L^{.138} , \tag{1}$$

$$z = \{D_p/[(1 + P/h)\sigma_L]\}^{.5} , \tag{2}$$

and

$$s_p = \mu_L + \sigma_L^{.832}(\sigma^2/\mu)^{.187}(.220/z + 1.142 - 2.866z) . \tag{3}$$

If $D_p/\mu > 1.5$, let $S = s_p + D_p$ and $s = s_p$. Otherwise, compute

$$S_o = \mu_L + v\sigma_L , \tag{4}$$

where v is the solution to

$$\int_{-\infty}^{v} \exp(-x^2/2)/\sqrt{2\pi}dx = p/(p+h) . \tag{5}$$

The policy parameters are given by

$$s = \text{minimum}\{S_p, S_o\} ,$$

$$S = \text{minimum}\{s_p+D_p, S_o\} .$$

If demands are integer valued, s_p, D_p, and S_o are rounded to the nearest integer.

1.3 Motivation for This Study

Ehrhardt [1] discussed how theoretical considerations lead to the following form for an approximately optimal policy,

$$D \doteq (2K\mu/h)^{.5} \tag{6}$$

and

$$s \doteq \mu_L + \sigma_L G(z) \ , \tag{7}$$

where

$$z \doteq D/[(1 + p/h)\sigma_L] \ , \tag{8}$$

and $G(\cdot)$ is a function that depends upon the demand distribution. The Power Approximation was derived by using these expressions as the bases of regression analyses, where known optimal policies are used as data. The regressions yielded parameter values in functions having the general forms (6), (7), and (8). The resulting expressions (1), (2), and (3) are further modified by (4) and (5) according to the empirical modification of Wagner [7]. In this study, we focus only on modifying (1), (2), and (3) to overcome several deficiencies in the original analysis.

The Power Approximation has three deficiencies. First, D_p in (1) converges to zero as the variance of demand goes to zero, whereas we would prefer that D_p converge to the Wilson Lot Size $D_w = (2K\mu/h)^{.5}$. Second, the approximation is not homogeneous with respect to the scaling of demand. Third, the numerical analysis conducted for fitting the approximation erred in a minor detail. These deficiencies are discussed in the following three sections.

1.3.1 *Limit of D*

The condition that $\lim D_p = D_w = (2K\mu/h)^{.5}$ as $\sigma^2 \to 0$, does not hold in (1) for the Power Approximation. In fact,

$\lim D_p = 0$ as $\sigma^2 \to 0$. This deficiency is remedied by using the following form

$$\hat{D}_p = a\mu^{(1-\beta)}(K/h)^{\beta}(1 + \sigma_L^2/\mu^2)^{\gamma} . \quad (9)$$

Our analysis, as described in the following sections, yields the result,

$$D_p = 1.30\mu^{.494}(K/h)^{.506}(1 + \sigma_L^2/\mu^2)^{.116} . \quad (10)$$

Hence,

$$\lim_{\sigma^2 \to 0} D_p = 1.30\mu^{.494}(K/h)^{.506} , \quad (11)$$

which is very close to the Wilson Lot Size

$$D_w = 1.414\mu^{.5}(K/h)^{.5} . \quad (12)$$

1.3.2 Non-homogeneity Deficiency

If demand is rescaled by a factor, say k, then s and D should be transformed similarly. That is, if $\hat{\mu} = k\mu$ and $\hat{\sigma} = k\sigma$, then we should have that $\hat{D} = kD$ and $\hat{s} = ks$. The Power Approximation, (1), (2), and (3), however, are not homogeneous. We remedy this situation in the present analysis as follows.

In [1], the regression for D_p used the model

$$\hat{D}_p = a\mu^{\alpha}(K/h)^{\beta}(\sigma_L)^{\gamma} . \quad (13)$$

In the current analysis, expression (9) is the form used; thus, we have replaced $(\sigma_L)^\gamma$ with $(1 + \sigma_L^2/\mu^2)^\gamma$. The 1 added to the last term ensures that $\hat{D}_p$ is not zero in the limit and the μ^2 divisor forces the last term to remain constant on a rescaling of demand. Requiring that $\alpha + \beta = 1$ forces homogeneity, since if we rescale demand by k, then the new mean is k times the old mean and the new holding cost is equal to the old holding cost divided by k.

Homogeneity was preserved in the fit for s by requiring a fit of the form in (7), where $G(\cdot)$ is dimensionless. This differs from the original Power Approximation, where σ_L was replaced with a different function of demand parameters.

1.3.3 Error in Numerical Analysis

In the original numerical analysis [1], three candidates for z in expression (3) were considered. They were

$$z_1 = (D_p/((p/h)\sigma_L))^{.5} , \tag{14}$$

$$z_2 = ((D_p + .5(\mu + \sigma^2/\mu^2))/((1 + p/h)\sigma_L))^{.5} , \tag{15}$$

and

$$z_3 = (D_p/((1 + p/h)\sigma_L))^{.5} . \tag{16}$$

Expression (16) was chosen over (14) and (15) because it provided the best numerical fit to optimal policy data. Inadvertantly, expression (15) was computed as

$$\hat{z}_2 = ((D_p + .5(1 + \sigma^2/\mu^2))/(1 + p/h)\sigma_L)^{.5} \tag{17}$$

in the original analysis. In the current analysis, the correct expression is considered. It does not, however, provide the best numerical fit with the data.

2. EXPERIMENTAL DESIGN

The data for this study are the same as those in the original analysis of the Power Approximation [1]. An experimental grid of 288 parameter settings is specified and presented in Table 1.

Three types of demand distributions are examined, namely, Poisson and Negative Binomial with variance-to-mean ratios of 3 and 9. Each demand distribution is evaluated with four mean values, 2, 4, 8, and 16. Lead time has three values, 0, 2, and 4. Since the cost function is linear in the parameters K, p, and h, the unit holding cost value can be normalized at unity. The unit penalty costs are 4, 9, 24, and 99, and the set up cost values are 32 and 64. The unit replenishment cost c need not be specified, since it does not affect the computation of an optimal policy for an undiscounted, infinite horizon model with complete backlogging. All combinations of the parameter settings are included in the grid, yielding 288 points.

The optimal policy for each of the 288 settings is calculated by the algorithm of Veinott and Wagner [7], implemented in the software written by Kaufman [4]. The resulting 288 values of s and S are the data utilized in the least squares regressions and the evaluation of results. The regression analysis is performed using the Statistical Package for the Social Sicences, second edition (1975).

TABLE 1

System Parameters

Factor	Levels	No. of Levels
Demand Distribution	Negative Binomial (Variance-to-Mean Ratio = 9)	3
	Negative Binomial (Variance-to-Mean Ratio = 3)	
	Poisson (Variance-to-Mean Ratio = 1)	
Mean Demand (μ)	2, 4, 8, 16	4
Replenishment Lead Time (L)	0, 2, 4	3
Replenishment Setup Cost (K)	32, 64	2
Unit Penalty Cost (p)	4, 9, 24, 99	4
Unit Holding Cost (h)	1	1

3. APPROXIMATIONS FOR D

Several mathematical forms were tested in seeking a new approximation for D. We considered a linear model

$$D = (2K\mu/h)^{.5} + [A + B\sigma/\mu + C\sigma^2/\mu^2] , \qquad (18)$$

and a multiplicative model

$$D = a\mu^{\alpha}(K/h)^{\beta}(1 + \sigma_L^2/\mu^2)^{\gamma}(p/h)^{\delta} . \qquad (19)$$

The best fits were obtained with the multiplicative form. Initial fits showed the variable p/h to be insignificant, so we arrived at the final model

$$D = a\mu^{\alpha}(K/h)^{\beta}(1 + \sigma_L^2/\mu^2)^{\gamma} , \qquad (20)$$

where we require $\alpha + \beta = 1$ (see Sec. 1.3.2).

Taking logarithms in (20), we have the linear expression

$$\ell n\, D_p = \ell n\, a + \alpha\, \ell n\, \mu + \beta\, \ell n\, (K/h) + \gamma\, \ell n\, (1 + \sigma_L^2/\mu^2) . \qquad (21)$$

We let $\alpha = 1 - \beta$, yielding

$$\ell n\, D_p = \ell n\, a + (1 - \beta)\, \ell n\, \mu + \beta\, \ell n\, (K/h) + \gamma\, \ell n\, (1 + \sigma_L^2/\mu^2) . \qquad (22)$$

The final form for fitting is

$$\ell n\, D_p - \ell n\, \mu = \ell n\, a + \beta[\ell n\, (K/h) - \ell n\, \mu] + \gamma\, \ell n\, (1 + \sigma_L^2/\mu^2) \qquad (23)$$

The result is

$$D_p = 1.30\mu^{.494}(K/h)^{.506}(1 + \sigma_L^2/\mu^2)^{.116} \qquad (24)$$

with $R^2 = .982$.

4. APPROXIMATIONS FOR s_p

The general form for s (from [1]) is

$$s_p = \alpha\mu_L + \sigma_L(A/z_i + B + Cz_i) . \tag{25}$$

Three alternative forms of z were considered:

$$z_1 = [D_p/((p/h)\sigma_L)]^{.5} , \tag{26}$$

$$z_2 = \left[\frac{D_p + .5(\mu + \sigma^2/\mu^2)}{(1 + p/h)\sigma_L}\right]^{.5} \tag{27}$$

$$z_3 = [D_p/((1 + p/h)\sigma_L)]^{.5} . \tag{28}$$

Each satisfies the assumptions of Roberts [6].

Model 1 (z_1).

Let

$$z = [D_p/((p/h)\sigma_L)]^{.5} . \tag{29}$$

Using least squares regression, we have

$$s_p = .973\mu_L + \sigma_L(.183/z + 1.063 - 2.192z) , \tag{30}$$

with $R^2 = .997$.

Model 2 (z_2) .

Let

$$z = [(D_p + .5(\mu + \sigma^2/\mu^2))/((1 + p/h)\sigma_L)]^{.5} . \tag{31}$$

Then least squares regression yields

$$s_p = .929\mu_L + \sigma_L(.190/z + 1.263 - 2.410z) \,, \tag{32}$$

with $R^2 = .998$.

Model 3 (z_3).

Let

$$z = [D_p/((1 + p/h)\sigma_L)]^{.5} \,. \tag{33}$$

Least square produces

$$s_p = .977\mu_L + \sigma_L(.171/z + 1.174 - 2.652z) + .36 \,, \tag{34}$$

which we call Model 3A; this model has $R^2 = .997$. We also consider a fourth rounded-down version, denoted Model 3B,

$$s_p = .977\mu_L + \sigma_L(.171/z + 1.174 - 2.652z) \,. \tag{35}$$

4.1 Cost Comparison and Choice of the Final Model

The 288 settings used in the regression analyses are examined first. The expected total cost per period is calculated for each item when controlled with each of the five policies: the optimal policy and the four policy generating models. All policy generating models evaluate D_p using expression (24), and s_p using expressions (30), (32), (34), and (35) for Models 1, 2, 3A, and 3B, respectively.

Let C(m), for m = 1, 2, 3A, 3B, and C* be the expected total cost per period for an item when controlled using a policy approximation (each of the four models) and the optimal policy, respectively. Our criterion of performance is

$$\Delta_p = \{[C(m) - C^*]/C\}*100\% , \tag{36}$$

which is the percentage by which the expected cost exceeds the optimal cost. The results for the 288 settings are summarized in Table 2, which lists the number of cases having values of Δ_p in various ranges.

Δ_p is displayed for the Power Approximation fcr comparitive purposes. Table 3 gives cumulative percentages.

Table 4 examines the characteristics of the settings which seem to be "outliers," that is, settings with Δ_p at least 3%.

The best of the four models seems to be Model 1. The average error of Model 1 is lowest. The number of items of low accuracy, for example, an error greater than 3.0%, is smallest. Finally, for a majority of the outliers, the error for Model 1 is less than or equal to that for the other models.

5. REVISED POWER APPROXIMATION

The algorithm for an approximately optimal (s,S) policy is as follows. Compute

$$D_p = 1.30\mu^{.494}(K/h)^{.506}(1 + \sigma_L^2/\mu^2)^{.116} \tag{37}$$

and

$$s_p = .973\mu_L + \sigma_L(.183/z + 1.063 - 2.192z) , \tag{38}$$

where

$$z = [D_p/((p/h)\sigma_L]^{.5} . \tag{39}$$

TABLE 2

ERROR FREQUENCIES

Model Δ_p	Power Approx.	1	2	3A	3B
[0.0.0.1)	151	118	117	108	116
[0.1,0.5)	102	108	107	110	105
[0.5,1.0)	21	25	31	31	32
[1.0,2.0)	11	18	11	16	15
[2.0,3.0)	3	11	13	14	12
[3.0,4.0)	0	4	4	6	2
[4.0,5.0)	0	3	3	2	4
[5.0,6.0)	0	1	2	1	2
Avg. Error	0.35%	0.469%	0.438%	0.507%	0.484%

TABLE 3

CUMULATIVE FREQUENCY (%) OF ERRORS

Model Δ_p	Power Approx.	1	2	3A	3B
[0.0,0.1)	52%	41%	40.6%	37.5%	40.3%
[0.1,0.5)	87.8%	78.5%	77.8%	75.7%	76.7%
[0.5,1.0)	95%	87.2%	88.5%	86.5%	86.5%
[1.0,2.0)	99%	93.4%	92.4%	92.0%	93.1%
[2.0,3.0)	100%	97.2%	96.9%	96.9%	97.2%
[3.0,4.0)	100%	98.6%	98.3%	99.0%	97.9%
[4.0,5.0)	100%	99.7%	99.3%	99.7%	99.3%
[5.0,6.0)	100%	100%	100%	100%	100%

TABLE 4

OUTLIERS

Item Characteristics						Error			
Setting Number	σ^2/μ	μ	p	K	L	1	2	3A	3B
12	1	16	24	32	0	3.4	3.4	3.4	3.4
16	1	16	99	32	0	3.9	3.9	3.8	3.9
17	1	2	4	64	0	1.3*	4.2	4.2*	1.3
193	9	2	4	32	0	4.5	4.5	4.5	4.5
205	9	2	99	32	0	5.7	5.7	5.7	5.7
206	9	4	99	32	0	3.2	5.2	3.2	5.2
221	9	2	99	64	0	3.4	3.4	3.4	4.0
237	9	2	99	32	2	4.3	4.3	3.7	4.3
253	9	2	99	64	2	4.1	3.6	3.6	4.1

*Not an outlier for this particular model.

If $D_p/\mu > 1.5$, then let $S = S_p + D_p$ and $s = s_p$. Otherwise compute

$$S_0 = \mu_L + v\sigma_L \ , \tag{40}$$

where v is the solution to

$$\int_{-\infty}^{v} \exp(-x^2/2)/\sqrt{2\pi}\, dx = p/(p+h) \ . \tag{41}$$

The policy parameters are

$$s = \text{minimum } \{S_p, S_0\} \ ,$$

and

$$S = \text{minimum } \{s_p + D_p, S_0\} \ .$$

If demands are integer valued, then s_p, D_p, and S_0 are rounded to the nearest integer.

5.1 Robustness of Revised Power Approximation

Here we examine parameter settings other than those used in deriving the approximation. Table 5 describes the settings for a full-factorial design with 32 cases, having both interpolated and extrapolated values for the parameters. With the exception of the unit holding cost, each parameter is set at two new values: one is an interpolation between levels in Table 1, and the other is an extrapolation beyond previous values in Table 1. Table 6 lists the frequencies of Δ_p for the 32 cases and also lists the results of the original Power Approximation. The costs for both approximations are only slightly higher than optimal; the costs for

TABLE 5

INTERPOLATED AND EXTRAPOLATED VALUES

Factor	Levels	No. of Levels
Demand Distribution	Negative Binomial Variance-to-Mean Ratio = 5 Negative Binomial Variance-to-Mean Ratio = 15	2
Mean Demand (μ)	0.5, 7.0	2
Replenishment Lead Time (L)	1, 6	2
Replenishment Setup Time (K)	16, 48	2
Unit Penalty Cost (p)	49, 132	2
Unit Holding Cost (h)	1	1

TABLE 6

ERROR FREQUENCY AND CUMULATIVE PERCENTAGE
FOR INTERPOLATED AND EXTRAPOLATED PARAMETER SETTINGS

Δ_p	Power Approximation		Model 1	
	Frequency	Percentage	Frequency	Percentage
[0.0%,0.2%)	12	38%	8	25%
[0.2%,0.6%)	8	61%	4	37.5%
[0.6%,1.0%)	4	75%	2	43.8%
[1.0%,2.0%)	3	84%	5	59.4%
[2.0%,3.0%)	3	94%	2	65.6%
[3.0%,4.0%)	1	97%	6	84.4%
[5.0%,11.0%)	1	100%	5	100%

the new approximation are slightly higher than those for the original Power Approximation. Of the eleven settings with values of Δ_p greater than 3.0%, two have two parameters with extrapolated values, four have four parameters with extrapolated values, and one has five parameters with extrapolated values.

We also examine the extreme extrapolations for individual parameter values. A base case is chosen for comparison. The parameter settings of the base case are near the midpoints of the ranges used in the 288 settings (negative binomial demand, $\sigma^2/\mu = 5$, $\mu = 9$, $L = 2$, $h = 1$, $p = 49$, and $K = 48$). The value of the variance-to-mean ratio, lead time, and penalty cost were extrapolated to 20, 10, and 199--slightly more than double the largest values used in the 288 settings. Table 7 lists the parameter settings and the resulting Δ_p for each of the extrapolations for both the Power Approximation and Model 1.

We see in all cases that the Power Approximation yields total costs within 1% of optimal, and Model 1 has only one case of slightly higher costs.

6. CONCLUSIONS

We have derived an approximately optimal policy that is easily computed, requires only the mean and variance of the demand distribution, and provides a good approximation to optimality over a wide range of parameter settings. The approximation is accurate when compared to optimal and is only slightly more costly than the original Power Approximation due to the added constraints ensuring homogeneity and the proper limit for approximation of D.

TABLE 7

SINGLE PARAMETER EXTRAPOLATIONS

Base Case: Negative Binomial Demand
(Variance-to-Mean Ratio = 5,
$\mu = 9$, $L = 2$, $p = 49$, $K = 48$)

Extrapolated Value	Power Approximation Δ_p	Model 1 Δ_p
Variance-to-Mean Ratio = 20	0.0%	1.06%
μ = 20	0.10%	0.01%
30	0.21%	0.14%
40	0.18%	0.16%
K = 20	0.11%	0.00%
15	0.28%	0.16%
9	0.63%	0.43%
P = 132	0.15%	0.02%
199	0.50%	0.18%
L = 10	0.02%	0.03%

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